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Parallel Line Microstrip Filters in an Inhomogeneous Medium

DOMINIQUE POMPEI, OLIVIER BENEVELLO, AND EDOUARD RIVIER

Abstract—**Parallel coupled microstrip lines in an inhomogeneous medium are studied. The quasi-static capacitance is shown to be linear with regard to the dielectric constant ϵ_r, simplifying the formalism used for analyzing microstrip filters.**

The electromagnetic advantages of the homogeneous medium carry over to the inhomogeneous medium. This result is obtained by equalizing all the velocities of the propagation modes.

I. Introduction

A NUMBER OF interesting properties have been found recently in the course of a study of microstrip filters. Some of these properties facilitate study of these filters; others should lead to new applications.

Microstrip filters in an inhomogeneous medium have not been studied as fully as stripline filters in homogeneous medium, in spite of important basic papers devoted to this subject [1]–[6]. The design of microstrip filters at frequencies above 1 GHz should take into account the existence of hybrid modes [7]–[9]. In order to reduce the problem at first, we made the usual assumption of a quasi-TEM mode. Higher order modes are introduced later, after the filter design using the quasi-TEM mode [7]–[9], [13].

II. Summary of the New Results

Many devices previously studied [3] have a set of N parallel coupled propagating lines. In microstrip technology, we are interested in a set of parallel microstrips deposited on a dielectric substrate. The metallized lower dielectric surface is grounded. The whole system is shielded by a conducting box (Fig. 1). Various filter configurations are shown in Fig. 2. Our approach [10]–[12] may be summarized as a) an extension of Kirchhoff's theory to a system of N parallel coupled transmission lines, and b) the introduction of boundary conditions, reducing the system to a two-port filter, whose response will be calculated.

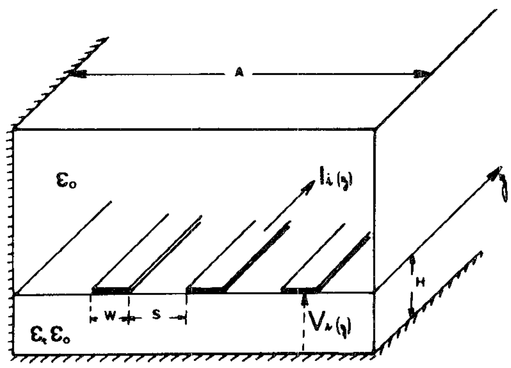

Fig. 1. Adjacent microstrip lines.

The new results may be summarized as follows.

1) Some parameters are perfectly linear with regard to the substrate ϵ_r. This point should allow easier studies for various values of ϵ_r.

2) A property of the various propagating modes allows the application of a new and simple formalism, making it possible to study filtering structures having a substantial number of lines coupled together.

3) The parallel microstrip structures in an inhomogeneous medium behave like structures in a homogeneous medium. The advantages of the homogeneous medium (such as width and regularity of the band) are maintained,

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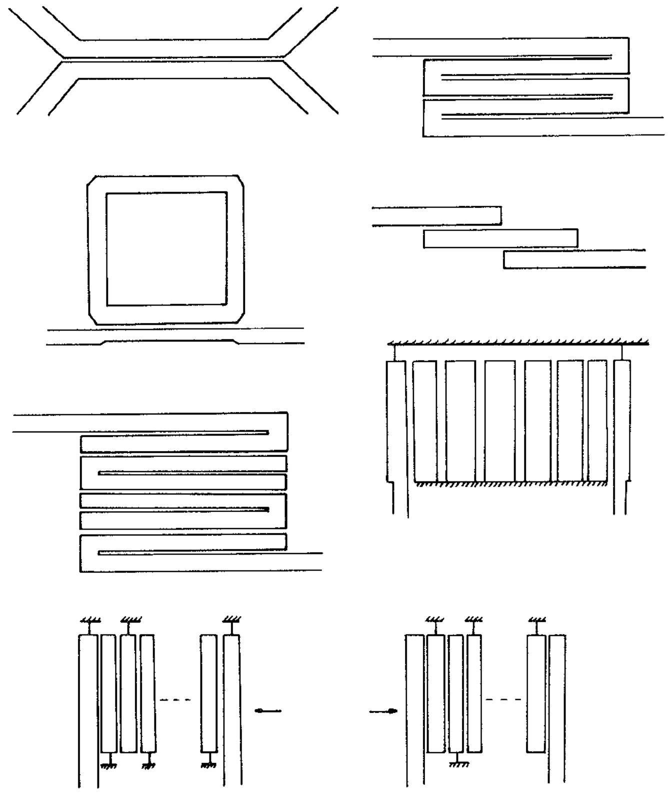

Fig. 2. Microstrip filters using parallel coupled lines.

and the advantages of the inhomogeneous medium (miniaturization) are also preserved.

III. Recapitulation of the General Formalism Constructed for Filter Analysis

A review of the earlier formalism [10]–[12] for quasi-TEM modes will first be given.

A system of N parallel microstrip lines, Fig. 1, is characterized by its matrix $[E]$ of eigen and mutual electrical influence coefficients

$$[E]=\begin{bmatrix} E_{11} & E_{12} & - - & E_{1N} \\ E_{12} & E_{22} & & \\ | & & \ddots & \\ E_{1N} & & & E_{NN} \end{bmatrix}$$

and by its matrix $[M]$ of eigen and mutual magnetic influence coefficients

$$[M]=\begin{bmatrix} L_{11} & M_{12} & - - & M_{1N} \\ M_{12} & L_{22} & & \\ | & & \ddots & \\ M_{1N} & & & L_{NN} \end{bmatrix}.$$

Let $I_i(z)$ and $V_i(z)$ $(i=1,\cdots,N)$ be the current in line i and the voltage between line i and the ground at the abscissa z.

Two equations in current and voltage may be formed.

$$\left(\frac{d^2}{dz^2}[U]+\omega^2[E][M]\right)\vec{I}(z)=0 \qquad (1)$$

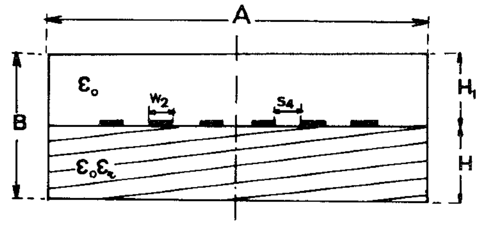

Fig. 3. Symmetrical adjacent microstrip lines.

$$\left(\frac{d^2}{dz^2}[U]+\omega^2[M][E]\right)\vec{V}(z)=0. \qquad (2)$$

$[U]$ is the unit matrix, ω is the angular frequency, and $\vec{I}(z)$ and $\vec{V}(z)$ are the vectors of components $I_i(z)$ and $V_i(z)$ $(i=1,\cdots,N)$.

Taking into account the formal analogy [11] of (1) and (2) with Lagrange's equations describing a coupled system having N degrees of freedom, it is possible to say that:

1) The set of N coupled lines has N propagating eigenmodes. There is a basis of eigenvectors in which (1) and (2) are uncoupled.

2) Each mode J is obtained by applying to each line i an eigenvoltage V_i^J (respectively, an eigencurrent I_i^J). The set of V_i^J $(i=1,\cdots,N)$ gives the eigenvoltage vector $\vec{V}^J$.

3) An eigenphase velocity v^J, identical on all the N lines, corresponds to the eigenvector $\vec{V}^J$.

If one puts

$$[G]=[E][M] \qquad (3)$$

the $\vec{V}^J$ and v^J are directly related to the eigenvectors and values of $[G]$.

Let $[E_o]$ be the matrix $[E]$ in the case, purely hypothetical, where the dielectric substrate is air $(\epsilon_r=1)$. It is known [14], [15] that, if the dielectric is not a magnetic one

$$[M]=\frac{1}{c^2}[E_o]^{-1}, \qquad c \text{ is light velocity in vacuum.} \qquad (4)$$

As a basic tool for explaining the improvements, we assume here the starting point of the previously published formalism. It is characterized by the following.

1) The acquiring of a technique of numerical computation for $[E_o]$ and $[E]$, for each given configuration of uniform parallel microstrip lines on a dielectric substrate and in a shielding box. This computation has been extended to adjacent devices having eight coupled lines by using a technique previously explained [16].

2) The determining of the eigenvectors and eigenvalues of the operator

$$[G]=\frac{1}{c^2}[E][E_o]^{-1}. \qquad (5)$$

The above formalism allows us to treat any configuration of coupled lines with different line widths w_i and different spacing between lines $S_{i,i+1}$.

In practice, this formalism is very difficult to apply when the device has no symmetry with regard to vertical mid plan (Fig. 3). It is much more difficult to apply when

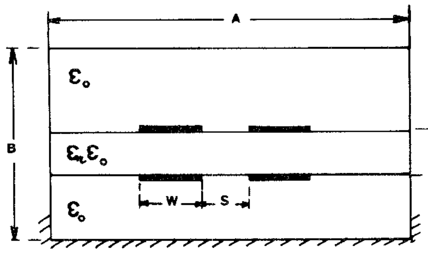

Fig. 4. Suspended microstrip lines.

the number of coupled lines is more than three. Indeed, the computation of the eigenvalues of $[G]$, absolutely any matrix, becomes more and more difficult.

The properties we found recently, and which are shown now, allow very important improvements in this formalism and particularly a very important simplification of the calculations.

IV. Main Properties Improving Formalism

The results affect largely the adjacent lines, but we also give information for suspended microstrip lines (Fig. 4).

A. Linearity in Terms of the Dielectric Constant

We studied the evolution of the matrix $[E]$ with regard to a variation of the relative dielectric constant ($\epsilon_r = 1$–70). This study is based on the numerical resolution of a large number of configurations with as many as eight coupled lines. We considered a wide range of parameters $A, B, H_1/H, W_i, S_{i,i+1}$ (Fig. 3), with asymmetrical structures ($W_i \neq W_j$ and $S_{i,i+1} \neq S_{j,j+1}$). Tight and low couplings are considered.

1) Property I: For a given configuration, the matrix $[E]$ is a linear function of the relative dielectric constant ϵ_r

$$[E]=(\epsilon_r-1)[A]+[E_o]. \tag{6}$$

The matrix $[A]$ depends on the dimensions of the structure but is independent of ϵ_r. This appears clearly on the graphs giving the absolute values for particular coefficients of the matrix $[E]$ versus ϵ_r for five (Fig. 5) and six lines (Fig. 6). Many similar results can be quoted.

It is important to note that, with this property, having calculated the electrical matrix $[E_o]$ in vacuum and $[E]$ for one substrate, we can calculate $[E]$ for any ϵ_r.

Property I was partially published for one or two coupled lines [17]. We have verified (6) for suspended microstrip coupled lines. We supply some data about this case (Fig. 7) but study of these structures must be extended.

2) The matrices $[E]$ and $[E_o]$ are real and symmetrical. Simple algorithms to diagonalize these matrices may be used. With such algorithms, we calculated the matrices $[E]_d$ and $[E_o]_d$ for many configurations. We have considered structures from two to eight adjacent coupled lines (Fig. 3) or from two to four suspended coupled lines (Fig. 4). They were calculated with many dielectric substrates and with tight couplings.

Property II: For a given configuration, the diagonalized matrix $[E_d]$ is a linear function of the dielectric relative constant ϵ_r.

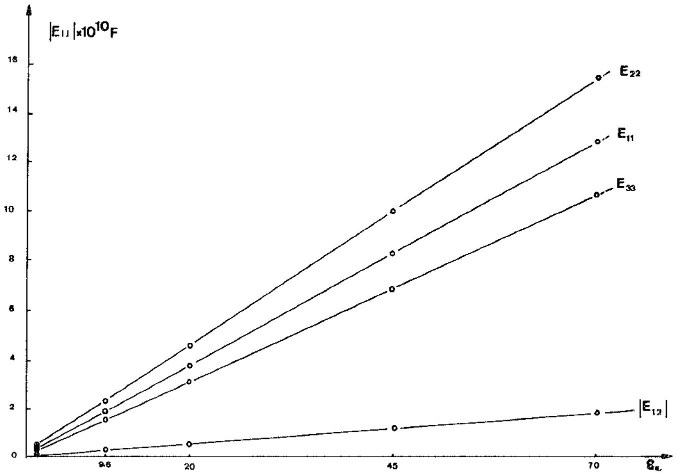

Fig. 5. Coefficients of the matrix $[E]$. Five adjacent microstrip lines. For geometrical dimensions, see Fig. 8.

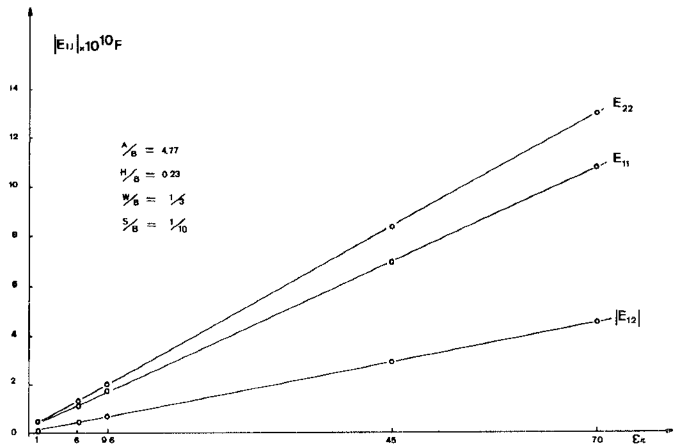

Fig. 6. Coefficients of the matrix $[E]$. Six adjacent microstrip lines.

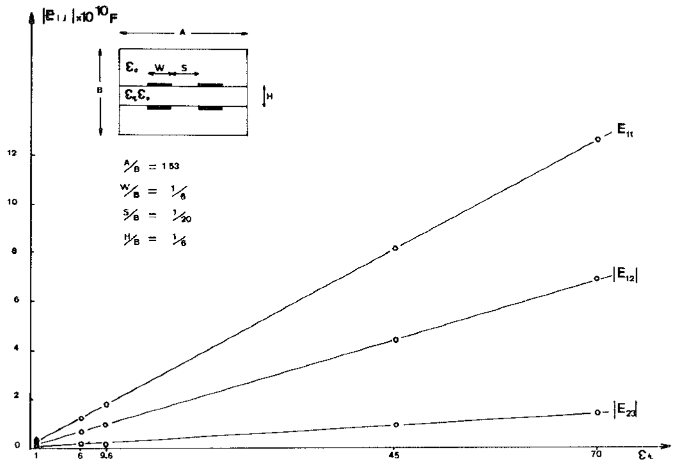

Fig. 7. Coefficients of the matrix $[E]$. Suspended microstrip lines.

$$[E]_d=(\epsilon_r-1)[A]_d+[E_o]_d. \tag{7}$$

The matrix $[A]_d$ depends on the structure dimensions, but not on ϵ_r. We give graphs of the coefficients of the

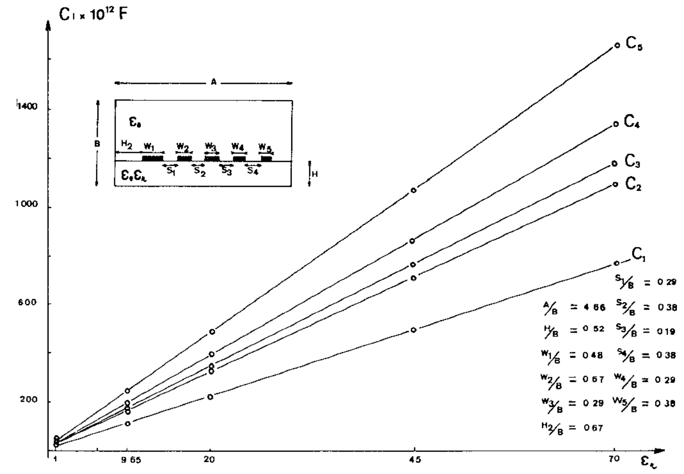

Fig. 8. Coefficients of the matrix $[E]_d$. Five microstrip lines.

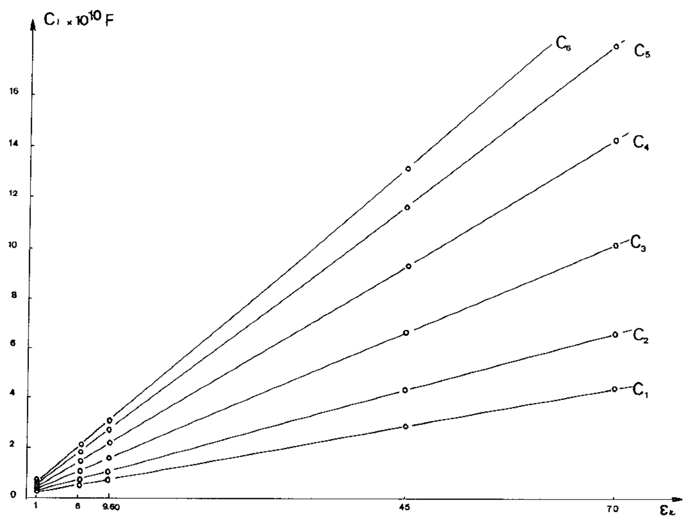

Fig. 9. Coefficients of the matrix $[E]_d$. Six microstrip adjacent lines. For geometrical dimensions, see Fig. 6.

matrix $[E]_d$ versus ϵ_r for five and six coupled lines (Figs. 8 and 9).

From the above results, having calculated the matrix $[E]_d$ for two particular values—one of which is $\epsilon_r = 1$—the general calculation of $[E]_d$ becomes very easy.

This property will be important for the calculation of the eigenmode velocities and for the improvement of the formalism.

B. Property III—Similarity of the Eigenvectors of $[E]$ and $[E_o]$

From the two properties previously given and especially the second one, we have shown that the matrices $[E]$ and $[E_o]$ have the same eigenvectors. With property II, we can write

$$C_i = (\epsilon_r - 1)a_i + C_{oi} \quad (i = 1, \cdots, N), \ N = \text{number of lines}. \tag{8}$$

But, we also have

$$i > j \Rightarrow C_i > C_j,\ a_i > a_j,\ C_{oi} > C_{oj} \qquad \text{(Figs. 8 and 9)}.$$

If T_N is the eigenvector of $[E]$ with the greatest eigenvalue C_N, we have

TABLE I
POTENTIAL (IN VOLTS) TO APPLY ON EACH LINE TO OBTAIN EIGENMODES

NUMBER OF COUPLED LINES	EVEN MODE		ODD MODE	
Adjacent Lines 4	+ 0.75	- 1	- 0.5	- 1
	+ 1	+ 0.75	+ 1	- 0.5
	+ 1	+ 0.75	- 1	+ 0.5
	+ 0.75	- 1	+ 0.5	+ 1
Adjacent Lines 5	+ 1	+ 0.7	- 1	+ 0.8
	0	+ 0.9	- 0.8	- 1
	- 1	+ 1	0	0
	0	+ 0.9	+ 0.8	+ 1
	+ 1	+ 0.7	+ 1	- 0.8
	- 0.4			
	+ 0.8			
	- 1			
	+ 0.8			
	- 0.4			
Suspended lines 4	+ 0.5	+ 0.5	- 0.5	- 0.5
	+ 0.5	- 0.5	+ 0.5	- 0.5
	+ 0.5	- 0.5	- 0.5	+ 0.5
	+ 0.5	+ 0.5	+ 0.5	+ 0.5
Suspended lines 3	+ 0.57	- 0.42	- 0.707	
	+ 0.58	+ 0.81	0	
	+ 0.57	- 0.42	+ 0.707	

$$\langle T_N | [E] T_N \rangle = C_N = (\epsilon_r - 1)a_N + C_{oN}.$$

If T_N is not an eigenvector of $[E_o]$ we have

$$\langle T_N | [E_o] T_N \rangle < C_{oN}$$

and with (6), we cannot obtain (8). Thus T_N must be an eigenvector of $[E_o]$. This demonstration is then repeated for $N-1$, and so on. As an example, we give eigenvectors of $[E]$ and $[E_o]$ for two adjacent microstrip structures and for two suspended microstrip structures (Table I).

V. Simplified Formalism for Studying Microstrip Filters

A. Velocities of Propagation and Characteristic Impedance Matrix

From property III, we can write many useful relations. If we note $[E]_d$ and $[E_o]_d$, the diagonalized forms of the matrices $[E]$ and $[E_o]$, we have

$$[E]_d = [T]^{-1}[E][T] \tag{9}$$

$$[E_o]_d = [T]^{-1}[E][T]. \tag{10}$$

So we obtain

$$[T]^{-1}[G][T] = \frac{1}{c^2}[E]_d[E_o]_d^{-1} = [G]_d. \tag{11}$$

Thus $[G]$ is diagonalizable and has the same eigenvectors as $[E]$ and $[E_o]$.

From $[G]_d$ we can easily calculate the velocities v^J of each propagating mode [12].

$$[v]=c\left[[E_o]_d[E]_d^{-1}\right]^{1/2} \tag{12}$$

with

$$[v]=\begin{bmatrix} v_1 & & 0 \\ & v_2 & \\ 0 & & \ddots \ v_N \end{bmatrix}.$$

So the determining of velocities is greatly improved.

Then, from property II, it is possible to calculate the velocities for all values of ϵ_r. This result is very important.

Similarly, $[T]$ being the diagonalization matrix of $[G]$, we can show that the eigencurrents $\vec{I}^J$ are the eigenvectors of $[E]$ and $[E_o]$.

For the determining of filters, we introduced [14] a characteristic impedance matrix $[Z_c]$ defined by

$$[Z_c]=\frac{[E]^{-1}[T][B][T]^{-1}}{\omega},$$

with ω = pulsation of the wave (13)

$$[B]^2=\omega[G]_d=\begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ \cdot & \beta_2 & & \\ \vdots & & \ddots & \\ 0 & & & \beta_N \end{bmatrix}^2.$$

$[Z_c]$ can be written in a diagonalized form

$$[Z_c]_d=\frac{\left[[E]_d[E_o]_d\right]^{-1/2}}{c}. \tag{14}$$

The elements Z_c^J of $[Z_c]_d$ are the characteristic impedances of the system, with as a particular case that of the symmetric coupler

$Z_c^{\mathrm{I}}=Z_{oe}$, even-mode characteristic impedance

$Z_c^{\mathrm{II}}=Z_{oo}$, odd-mode characteristic impedance.

B. Application of Property III to N Coupled Lines

Equations (1) and (2) have solutions [10] which are generalizations of Kirchhoff's relations for a single line. From these equations, we have written the general form of the impedance matrix $[Z]$ for a system of N coupled lines of length L.

From the relations given by property III, we can obtain a simpler formalism

$$[Z]=[T_Z]\begin{bmatrix} -[Z_c]_d & 0 \\ 0 & [Z_c]_d \end{bmatrix}[T_Z]^{-1} \tag{15}$$

with

$$[T_Z]=\begin{bmatrix} [T] & 0 \\ 0 & [T] \end{bmatrix}\begin{bmatrix} [U] & [U] \\ \exp(j[B]L) & \exp(-j[B]L) \end{bmatrix} \tag{16}$$

where $[U]$ is the unit matrix, and

$$\exp(j[B]L)=\begin{bmatrix} \exp(j\beta^{\mathrm{I}}L) & \cdots & 0 \\ 0 & \exp(j\beta^{\mathrm{II}}L) & \\ \vdots & & \ddots \\ 0 & & \exp(j\beta^N L) \end{bmatrix}.$$

C. Application to Filters

If we know the $N\times N$ matrix $[Z]$ defined by

$$\begin{bmatrix} \vec{V}(0) \\ \vec{V}(L) \end{bmatrix}=|Z|\begin{bmatrix} \vec{I}(0) \\ \vec{I}(L) \end{bmatrix}$$

where $\vec{V}(z)$ and $\vec{I}(z)$ are the voltage and current vectors at the abcissa z (see (1) and (2)). We only have to apply at the ends $z=0$ and $z=L$ (Fig. 1) any adequate boundary condition to construct a filter like those shown in Fig. 2.

$[Z]$ degenerates in a 2×2 impedance matrix $[Z]'$ of a two-port system. Well-known transformations allow us to obtain the scattering matrix $[S]$, then the wanted transfer function.

Important Note: In the whole study it is found that to obtain the transfer function, the only fundamental operations are to calculate the eigenvalues and vectors of $[E]$ and $[E_o]$.

The above procedure being very simple, it is possible to provide a looped program on a computer giving, by successive corrections, a filter corresponding to a given pattern. So analysis and synthesis may be coupled.

VI. Remarks on the Above Properties

As we noted before, properties I and II resulted from a systematic study of many configurations. This fact allowed us to estimate the precision of results.

We noted therefore that the linearity of $[E]$ as a function of the dielectric constant ϵ_r was well proved for the coefficients of $[E]$ such as $E_{i,i}$, $E_{i,i+1}$, $E_{i,i+2}$. This fact is less evident for the other coefficients $E_{i,j}$ ($j\geqslant i+3$), but it is a consequence of the precision of the method; a change in the mesh [12] could improve the result if necessary.

On the other hand, for the linearity of $[E_d]$ as a function of ϵ_r, it has been very well proved in the various cases studied with configurations having asymmetries in the cross section.

This comes from the fact that diagonal terms of $[E]$ are much larger than terms such as $E_{i,i+3}\cdots$.

Finally, it can be noted that the first two properties have been established by calculation on a computer as opposed to property III which has been mathematically proved from the two previous ones.

VII. Equalization of Eigenmode Phase Velocities

A. Geometrical Method

Let us consider a configuration with $H_1=H$ (Fig. 10).

As $[A]$ is independent of the dielectric constant, it is

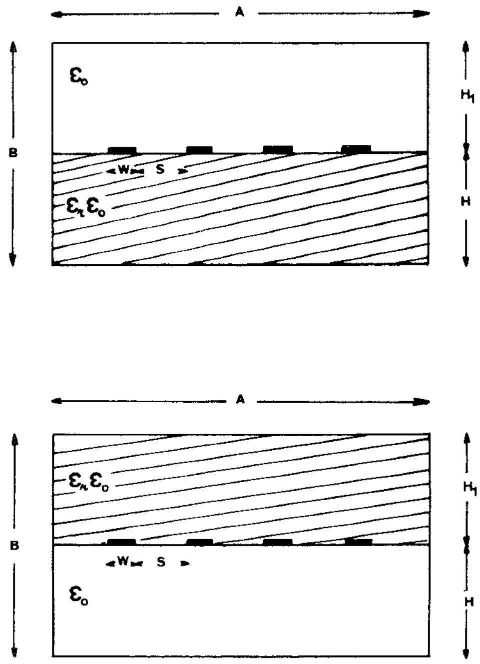

Fig. 10. Adjacent microstrip lines with $H_1 = H$.

unchanged when the positions of the dielectric and the air are interchanged.

In the first case (Fig. 10(a)) we have

$$[E]=(\epsilon_r-1)[A]+[E_o].$$

In the second case (Fig. 10(b)) we have

$$[E]'=(\epsilon_r-1)[A]+[E_o].$$

If we superimpose the two cases, we get

$$[E]+[E]'=2(\epsilon_r-1)[A]+2[E_o]=(\epsilon_r+1)[E_o]$$

because it is a similar process of superposing a homogeneous configuration filled with air ($\epsilon_r = 1$), and a homogeneous configuration filled with a dielectric ϵ_r. So it is inferred that

$$\text{if } H_1 = H, \qquad \text{then } [A]=\tfrac{1}{2}[E_o]. \tag{17}$$

Let us consider now (12)

$$[v]=c\left[[E_o]_d[E]_d^{-1}\right]^{1/2}.$$

From (17) we get

$$[E]=\frac{\epsilon_r+1}{2}[E_o], \qquad \text{then } [E]_d=\frac{\epsilon_r+1}{2}[E_o]_d.$$

So we note that

$$[v]=\left(\frac{\epsilon_r+1}{2}\right)^{1/2}c[U], \qquad [U] \text{ being the unitary matrix.} \tag{18}$$

Therefore, if $H_1 = H$, whatever the number of coupled lines and whatever the geometrical configuration, all the propagation velocities of various modes are equal. This property appears to be very interesting because it induces a further simplification of the previous formalism; indeed, we obtain in this case the same properties as for the homogeneous case.

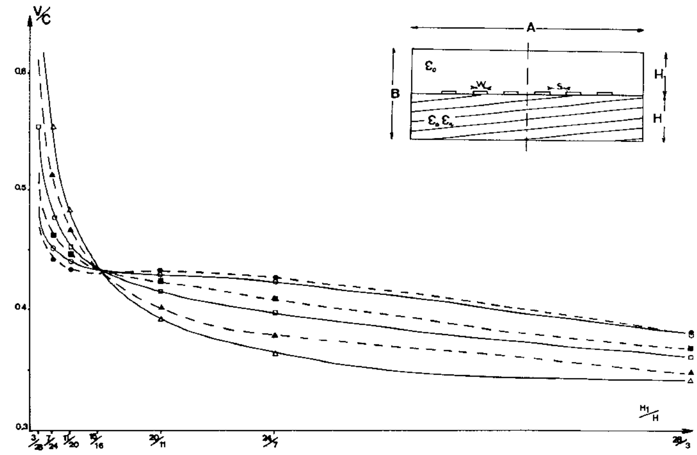

Fig. 11. Velocities of propagation. Six symmetrical adjacent microstrip lines. $\epsilon_r = 9.6$. - - - - - - Odd mode. ——— Even mode.

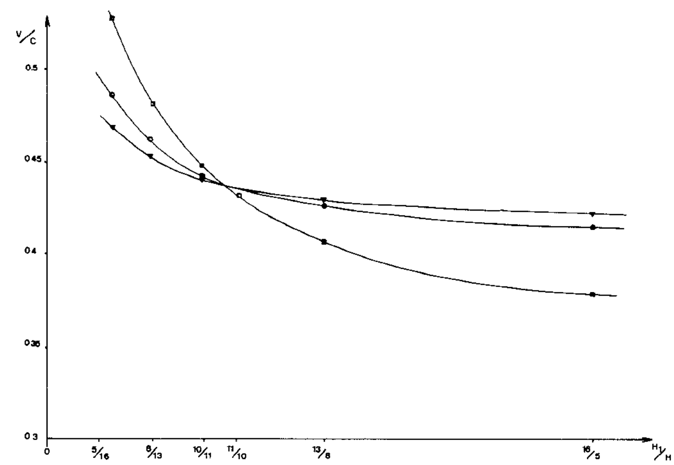

Fig. 12. Velocities of propagation. Three symmetrical adjacent lines. $\epsilon_r = 9.6$. ∇Even mode. ■Even mode. ●Odd mode.

Thus for example, we know that the directivity of couplers in an inhomogeneous medium is generally bad; the attenuation of the directivity with increasing frequencies comes from the divergence between the two propagation velocities of even and odd modes.

In the case $H_1 = H$ we will obtain a very interesting improvement.

Furthermore, we note that this equalization of velocities has been previously considered after experimental results by some authors [18]–[20], but it remained to our knowledge only a supposition.

We give some graphs describing the normalized mode velocity as a function of H_1/H. On these graphs the equalization of velocities, when $H_1 = H$, is easily seen (Figs. 11 and 12).

B. Equalization of Phase Velocities with an Overlay Method

Another method can be used for equalizing the velocities; it consists of coating the strip with a thin layer of

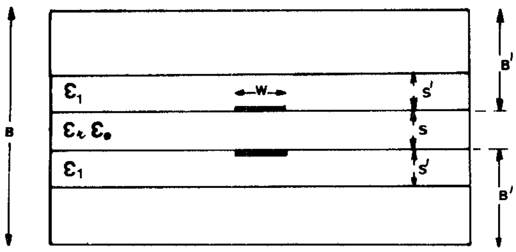

Fig. 13. Microstrip suspended lines with overlay dielectric.

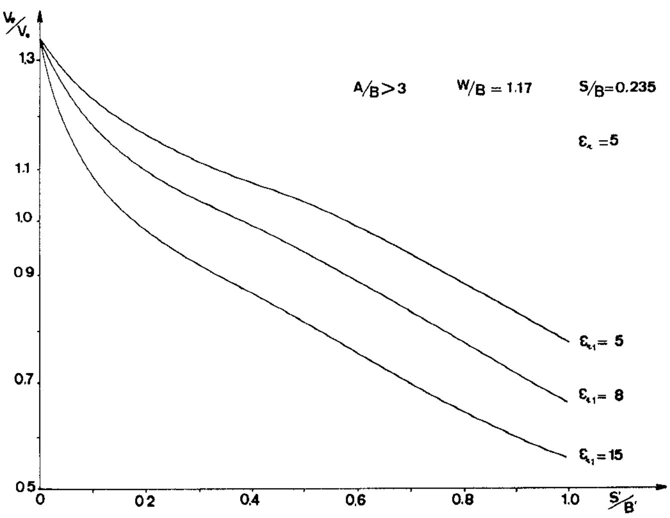

Fig. 14. $= V_e / = V_o$ for two suspended microstrip lines.

dielectric substrate with a larger ϵ_r (Fig. 13). This technique allows us to obtain a matrix $[E]$ equivalent to the matrix of a homogeneous medium, because the constant ϵ_{eff} of the propagation medium is greatly altered.

The equalization of the velocities can be obtained either by varying the thickness of the layer for a fixed value of the permittivity, or by varying the dielectric constant of the overlay for a fixed value of its thickness (Fig. 14) for suspended microstrip lines.

C. Characteristic Impedances

In this study we were led to calculate the electrical characteristics of many coupled line systems. So we have plotted many graphs giving the characteristic impedance of each propagating mode as a function of geometrical dimensions.

We give here graphs for a coupler in the adjacent technique, with an equalization of velocities ($H_1 = H$) (Fig. 15) and for a three coupled line device (Fig. 16).

In this last case, it is evident that there are two even-propagating modes and an odd one. We can provide many other similar graphs.

VIII. Conclusion

New results about N coupled microstrip lines in an inhomogeneous medium appear in this paper. The eigenmodes of propagation are well acquired; their velocities

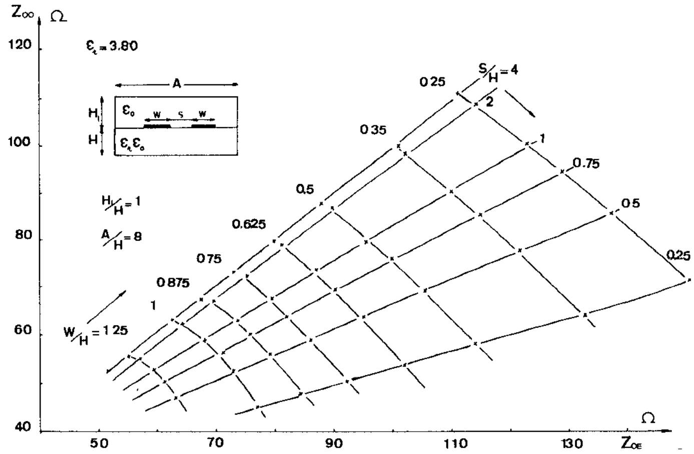

Fig. 15. Z_{oo} versus Z_{oe} for two adjacent microstrip lines with $H_1 = H$.

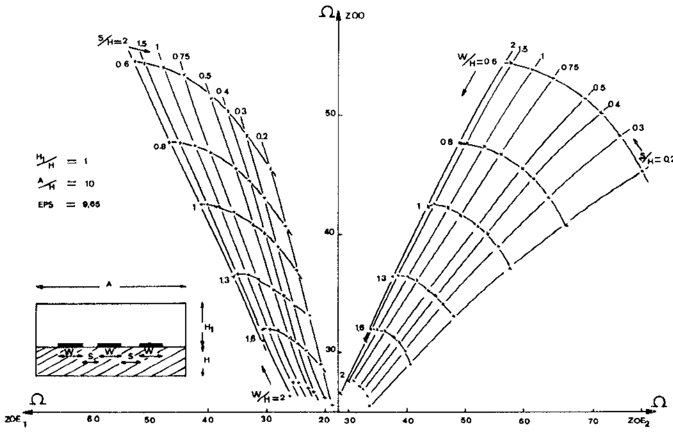

Fig. 16. Z_{oo} versus Z_{oe} for three adjacent microstrip lines with $H_1 = H$.

are easily calculated. The problem is simplified with the linearity of the matrices $[E]$ and $[E]_d$ versus ϵ_r.

All these properties improve the theory of the microstrip filters. In particular, the equality of the eigenvelocities, when the dielectric substrate and the vacuum have the same thickness, is clearly demonstrated.

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The Characteristic Impedance of Rectangular Transmission Lines with Thin Center Conductor and Air Dielectric

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Abstract—**The characteristic impedance of large-scale rectangular strip transmission line facilities used for such purposes as EMI susceptibility testing, biological exposures, etc., is discussed. These lines are characterized by a thin center conductor and an air dielectric. Impedance data obtained by earlier workers, using different analytical and numerical techniques, are reviewed and compared. Exact data are available for the problem involving a center conductor of zero thickness, while for the center conductor of finite thickness, data are available which are accurate to less than 1.25 percent.**

I. INTRODUCTION

RECTANGULAR COAXIAL transmission lines which contain a propagating transverse electromagnetic (TEM) field are finding increasing application in such areas as EM susceptibility and emissions testing, biological effects of RF exposure, and calibration of radiation survey meters and electric field probes. Such lines possess an air dielectric with a thin center conductor, thereby maximizing the test space available between conductors. Crawford [1] has discussed the properties of such lines as well as their advantages, and has described a family of TEM "cells" constructed at the National Bureau of Standards. A similar transmission line of this type, used for purposes of exposing monkeys as well as large phantoms to HF-band (10–30-MHz) radiation fields has also been described [2], [3]. Others [4] have used much smaller rectangular lines of this type to investigate the interaction of microwaves with isolated nerve cells at frequencies up to 3 GHz. The use of such lines for calibration of radiation survey (hazard) meters as well as electric and magnetic field probes in the VHF and UHF bands has been discussed by Crawford [5], Baird [6], and Aslan [7]. A series of these transmission lines is now manufactured commercially by Instruments for Industries, Inc., Farmingdale, NY, and has been termed "Crawford Cells" by the manufacturer.

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The characteristic impedance of such transmission lines has been quoted by Crawford [1] in terms of the fixed dimensions of the line's cross section (see Fig. 1 for notation) as well as an unknown fringing capacitance per unit length C_f'.

$$Z_0 = \frac{376.73}{4\left[w/(b-t) + C_f'/\epsilon\right]} \cdots \tag{1}$$

where $\epsilon = 8.8542 \times 10^{-12}$ F/m, assuming an air dielectric. Crawford used time-domain reflectometry methods to ex-